faculty of science
department of mathematics

Final Examination

MATH 232 D100 Spring 2012

Instructor: D. J. Katz

April 16, 2012, 8:30 a.m. – 11:30 a.m.

Name: ______________________________ (please print)
family name *given name*

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student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. To receive full credit for a particular question you must provide a complete and well presented solution.
5. This exam has 11 questions on 11 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. Leave answers in "calculator ready" expressions: such as $3 + \ln 7$ or $e^{\sqrt{2}}$.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, or copying from, other examinees is forbidden.**

Question	Maximum	Score
1	8	
2	7	
3	12	
4	8	
5	8	
6	7	
7	13	
8	7	
9	9	
10	14	
11	11	
Total	104	

1. (The following three parts are unrelated to each other.)

[2] (a) Let $A = \begin{pmatrix} -5 & 2 & 1 \\ -4 & 1 & 3 \\ 1 & 0 & 3 \end{pmatrix}$. Find $A + A^T$.

[3] (b) Let $B = \begin{pmatrix} 1 & 3 & -3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$. Find $\det(B^3)$.

[3] (c) Express $e^{5\pi(1+i)/2}$ in the form $a + bi$, where a and b are real numbers.

2. Let $A = \begin{pmatrix} 0 & 1 & -3 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{pmatrix}$.

[3] (a) Find a basis of the row space of A.

[2] (b) What is the rank of A? Justify your answer.

[2] (c) What is the dimension of the null space of A? Justify your answer.

3. Consider the linear operator T such that $T(1,0,0) = (1,2,0)$, $T(0,1,0) = (1,1,-1)$, and $T(0,0,1) = (2,1,-3)$.

[3] (a) Find the standard matrix for T, that is, find the the matrix $[T]$ such that $T(\mathbf{x}) = [T]\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

[2] (b) Find $T(-1,2,1)$.

[3] (c) What is the kernel of T? If the kernel has dimension 1 or higher, give a basis for it.

[2] (d) Is T a one-to-one transformation? Explain why or why not.

[2] (e) Is T an onto transformation? Explain why or why not.

[5] **4.** (a) Find the standard matrix for counterclockwise rotation by $\pi/3$ followed by reflection across the y-axis in $\mathbb{R}^2$.

[3] (b) The matrix $\begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ is either a rotation or a reflection in $\mathbb{R}^2$. Which one is it? Justify your answer.

[8] **5.** Only one of the three sets B_1, B_2, and B_3 given below is a basis of the plane $x+y+2z=0$ in $\mathbb{R}^3$. Which one is it? You must explain why the one you pick is a basis, and you must explain why the other two are not.

$$B_1 = \{(1,1,-1),(1,1,0)\}$$
$$B_2 = \{(1,-1,0),(2,2,-2)\}$$
$$B_3 = \{(1,1,-1),(2,2,-2)\}$$

[2] **6.** (a) If M is a 3×3 matrix, what are the possible values for the rank of M? List all possible values. (And do not list impossible values.)

[5] (b) The rank of $A = \begin{pmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{pmatrix}$ depends on the value of t. For each of the possible values of $\operatorname{rank}(A)$, find the values of t (if any) that make $\operatorname{rank}(A)$ equal to that value.

7. Let W be the plane in $\mathbb{R}^3$ with equation $x + 2y - z = 0$.

[2] (a) What sort of geometric object is $W^{\perp}$? Justify your answer.

[2] (b) Find a basis for $W^{\perp}$.

[5] (c) Find the orthogonal projection of the vector $\mathbf{v} = (1, 4, 0)$ onto W and the orthogonal projection of $\mathbf{v}$ onto $W^{\perp}$.

[2] (d) What is the point in W closest to $(1, 4, 0)$? Justify your answer.

[2] (e) What is the distance from the point $(1, 4, 0)$ to the plane W?

8. Let $W = \text{span}\{(1,1,0),(3,0,-1)\}$.

[2] (a) Show that $\{(1,1,0),(3,0,-1)\}$ is not an orthogonal basis of W.

[3] (b) Find an orthogonal basis of W.

[2] (c) Find an orthonormal basis of W.

9. Let $A = \begin{pmatrix} 7 & 8 \\ -4 & -5 \end{pmatrix}$.

[3] (a) Show that -1 and 3 are eigenvalues of A, and that the corresponding eigenvectors are $(1, -1)$ and $(2, -1)$, respectively.

[3] (b) Find matrices P and Λ such that Λ is diagonal and $A = P\Lambda P^{-1}$. (That is, diagonalize A.)

[3] (c) Find a diagonal matrix D such that $A^{57} = PDP^{-1}$, where P is the same as in part (c). You can express the diagonal entries of D using exponential notation: do not calculate out the actual values.

10. Let $A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

[3] (a) Find the eigenvalues of A.

[2] (b) For each eigenvalue of A, find its algebraic multiplicity.

[6] (c) For each eigenvalue of A, find its geometric multiplicity.

[3] (d) Explain why A is not diagonalizable.

11. The characteristic polynomial of a matrix A is $\lambda^4(\lambda-1)^2(\lambda-3)(\lambda+5)^3$.

[2] (a) How many rows does A have? How many columns does A have?

[3] (b) What is the trace of A?

[3] (c) Is A invertible?

[3] (d) Does A have any fixed points other than $\mathbf{0}$?

(You must justify all your answers in order to get credit.)